Stefan-Boltzmann's law

Number	133720-EN	Topic	Light, IR, energy of electromagnetic radiation				
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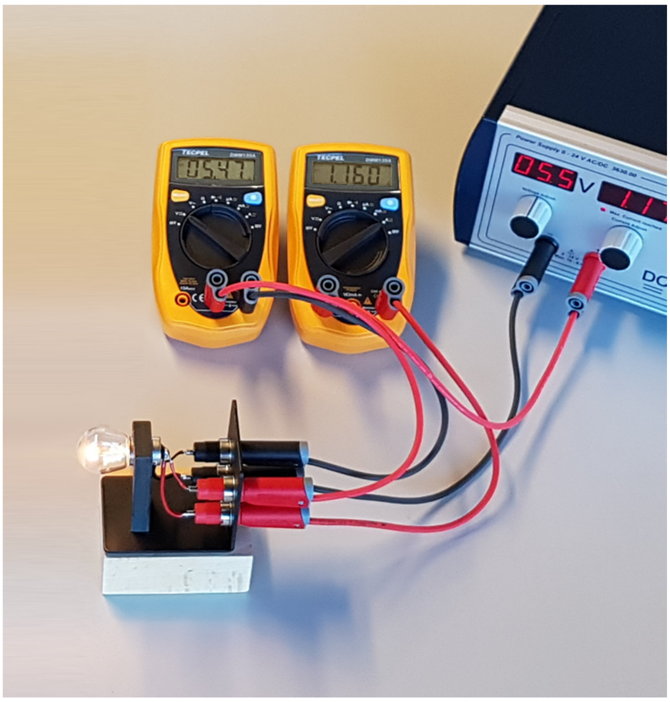


Objective
To investigate Stefan-Boltzmann's law:
The radiated power of an absolute black body is proportional to the absolute temperature to the fourth power.

Principle
The tungsten filament in an incandescent light bulb behaves approximately as a black body at the relevant temperatures.

The temperature of the filament is determined from its resistance.

The radiated power is found from the input electric power and a model for the loss by thermal conduction.

Equipment
Stefan-Boltzmann's lamp
Resistor (100 Ohm)

(Indoor-) thermometer

Power supply
Multimeters (two)
Lab leads

Stefan-Boltzmann's law
Josef Stefan derived the radiation law empirically in 1879 and Ludwig Boltzmann was in 1884 able to give the law a theoretical foundation in thermodynamics.

An ideal black body with area A and temperature T emits the power P_R given by:

$$P_\mathrm{R} = A \cdot \sigma \cdot T^4$$

where σ is the Stefan-Boltzmann constant. I this experiment we investigate the T^4 dependency.

As the area is constant, we can combine the two constants:

$$P_\mathrm{R} = s \cdot T^4$$

Measurement principles

Determining the temperature

The filament is made from the metal tungsten. The resistance in a wolfram wire depends on its temperature in a well-defined way. The resistance is found using Ohm's law.

To calculate the temperature of the filament we need the value of the resistance R_{REF} at room temperature t_{REF}. (This cannot be done by a single measurement as the measurement current will cause a temperature rise – the method is described below.)

When we know the room temperature t_{REF} and the corresponding resistance R_{REF}, there is a formula to find the temperature of the filament from its resistance – see details in *Calculations*.

The energy balance

In a static situation, the thermal energy is constant. The input electric energy goes to radiated light as well as heat loss (i.e. *conduction* through the leads and *convection* in the gas in the bulb):

$$P_{\mathrm{EL}} = P_{\mathrm{R}} + P_{\mathrm{LOSS}}$$

Low temperature: Radiated power almost zero

At lower temperatures (where $P_R \approx 0$), the electric power will balance the loss:

$$P_{\mathrm{EL}} \approx P_{\mathrm{LOSS}}$$

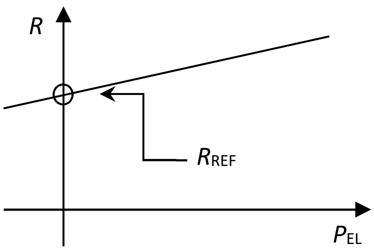
We will assume that the loss by conduction and convection is proportional with the temperature difference from the filament to the ambient:

$$P_{\mathrm{LOSS}} = K_{\mathrm{LOSS}} \cdot (T - T_{\mathrm{AMB}})$$

Near room temperature the filament resistance is a linear function of its temperature.

This means that the resistance all in all is a linear function of the input electric power in this temperature range. It is thus possible to find R_{REF} from a graph of resistance versus input power P_{EL}:

See sketch. Where $P_{EL} = 0$, heating is zero as well:



Now we have the information needed to find the temperature of the filament based on its resistance. Next, the same measurements we used for R_{REF} can be used for determining K_{LOSS}.

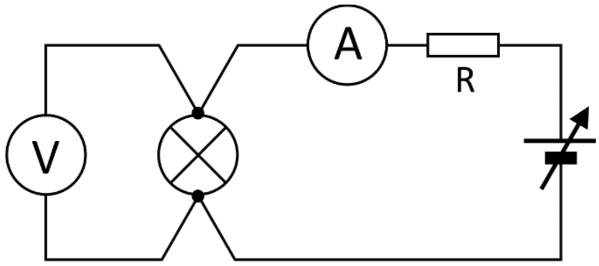
This enables us to calculate P_{LOSS} – and hence P_R – for all temperatures.

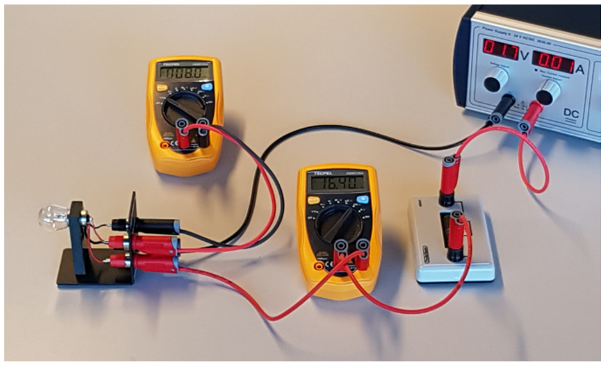
Procedure

Measurements must be precise – use external instruments instead of the build-in meters on the power supply.

1 – Determining t_{REF}, R_{REF} and K_{LOSS}

Rather small voltages and currents are used. In order to facilitate adjustment of the power supply, a series resistor is inserted – see schematics and photo below.





During the first measurements a resistor is placed in series connection with the lamp

Note that the voltmeter is plugged into the upper two sockets alone in order to measure the voltage U directly across the incandescent lamp.

Set the voltmeter for mV and the ammeter for mA.

Increase the voltage carefully, until the voltmeter reads about 25 mV and read the precise value of voltage and current. Repeat for 40, 55, 70 85 mV.

Determine the room temperature t_{REF}.

Write down the results in a table like this:

t_{REF} :		°C

U	I
mV	mA

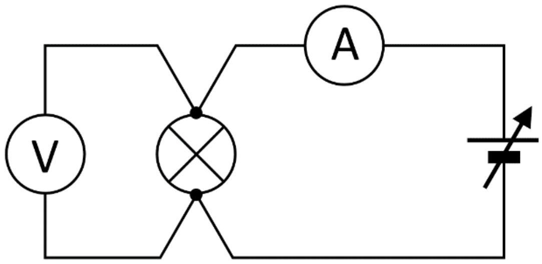
2 – Higher temperatures

Note: The resistor is only used in part 1. It must be removed for the rest of the experiment. (Schematics to the right, photo on front page.)

Caution: Set the ammeter for A – usually another socket than for mA!

Measure lamp voltage U and current I, starting with U = 0.5 V. Increase the voltage about 30 – 40 % for each measurement. Last measurement is with 12 V. (This makes 10 to 12 measurements.)

Write the results down in a table.



Calculations

Using a spreadsheet is highly recommended. It will need at least the columns shown below. Reserve cells for the values of T_{AMB} (K), R_{300} (Ω) and K_{LOSS} (W/K).

U	I	R	R / R_{300}	T	P_{EL}	P_{LOSS}	P_R
V	A	Ω		K	W	W	W

1a – Determining R_{REF} and t_{REF}

Calculate resistance R and input power P_{EL} from the voltage and the current. Plot R as a function of P_{EL} as described in *Measurement principles* and draw a straight line.

Read R_{REF} where the line crosses the R axis.

Now, we are ready to calculate the temperature.

In order to use a *universal* expression for temperature, we must first from R_{REF} find the resistance at some standard temperature, chosen as 300 K:

Based on a table value of the temperature coefficient of tungsten near room temperature, we can derive the following formula giving R_{300} from the measurement at room temperature t_{REF} (unit: °C):

$$R_{300} = R_{\mathrm{REF}} \cdot \left(1 + \frac{26{,}8\,°\mathrm{C} - t_{\mathrm{REF}}}{208{,}3\,°\mathrm{C} + t_{\mathrm{REF}}}\right)$$

From the actual resistance R of the filament, its absolute temperature T is found by first calculating the relative resistance change $x = R/R_{300}$ which is then inserted into a sixth order polynomial:

$$T = a_6\,x^6 + a_5\,x^5 + a_4\,x^4 + a_3\,x^3 + a_2\,x^2 + a_1\,x + a_0$$

The coefficients are given by:

a_6 :	$-9{,}915 \cdot 10^{-5}$	a_2 :	-19,76
a_5 :	$6{,}794 \cdot 10^{-3}$	a_1 :	267,3
a_4 :	-0,1840	a_0 :	52,38
a_3 :	2,520		

1b – Determining K_{LOSS}

For the results in part 1, P_{EL} is now plotted as a function of $(T - T_{AMB})$. Draw the best straight line through the data points and find K_{LOSS} as the slope of the line.

2 – Higher temperatures

For measurements in part 2, calculate P_{LOSS} from the value of K_{LOSS} just found. Calculate P_R also.

Note: This is done **only** for measurements in part 2.

Make a log-log plot of P_R as a function of T. We expect a dependency as $P_{\mathrm{R}} = s \cdot T^4$, so the result should appear as a straight line.

On a log-log paper, the exponent is found as the slope.

In a spreadsheet, both axes are formatted as logarithmic, preferably with base 2, and a "trend line" of type *Power*. Let the equation be displayed – then the exponent can be read directly.

Extra problem

Assume that the filament is an ideal black body and that its total length is 50 mm (it is curled up in a helix – check with a magnifying glass). Look up Stefan-Boltzmann's constant.

What is the diameter of the filament? (You need to find the value of s first.)

Discussion and evaluation

Can the emission from the reasonably well be described with the expression $P_{\mathrm{R}} = s \cdot T^4$?

Can you explain possible deviations?

(If you answered the *Extra problem*:
Is the value for the thickness reasonable?)

Teacher's notes

Concepts used

Energy and power
Absolute temperature
Ohms law
Electric power

Mathematical sills

Linear functions
Using a spreadsheet (above beginner's level)

About the equipment

One could imagine measuring the radiated power directly by means of a broad band detector. This will give fairly acceptable results, but they will suffer from absorption of infrared radiation by the glass of the lamp. This means too low readings for lower temperatures, typically resulting in an exponent markedly larger than 4.

The method described in this manual avoids this problem.

The filament is of course not an ideal black body. But it behaves reasonably well as a "grey body", meaning that it absorbs all wavelengths equally well, even if the absorption is less than 100 %. Stefan-Boltzmann's law can then be corrected by introducing the emissivity ε :

$$P_{\mathrm{R}} = \varepsilon \cdot A \cdot \sigma \cdot T^4$$

The factor $\varepsilon \cdot A \cdot \sigma$ is still a constant. In this experiment the area A only enters the game in the very last extra question, and it is not important that ε is not exactly equal to 1.

On our home page, you can be download a spread-sheet with the calculations of temperature from resistance. Results (or the formulae) can then be copied to the main spread sheet.
(search for Item No. 277022.)

Detailed equipment list

Specifically for the experiment

277022	Stefan–Boltzmann's lamp
420541	Resistor 100 Ohm, 10 W, 1 %

Standard lab equipment

361600	Power supply (or similar)
386135	Multimeter (or similar) (Qty. 2)
105720	Safety cable, silicone, 50 cm black (Qty. 2)
105721	Safety cable, silicone, 50 cm, red (Qty. 3)