Newton's second law

Experiment number	134710-EN	Topic	Mechanics				
Version	2017-08-30 / HS	Type	Student exercise	Suggested for	grade 11-12+	p.	1/4

Objective

An experimental demonstration of Newton's second law.

Principle

A cart (glider) on a horizontal air track accelerates using a thread, a pulley, and a weight influenced by gravity. The mass of the weight and the cart are weighed.

At two different positions, photogates measure the speed of the cart as well as the time interval spent between the two photogates. The actual acceleration of the cart is calculated from these measurements.

Newton's second law leads to a theoretical value for the acceleration which is compared to the measured one.

Equipment

(Detailed equipment list on p. 4)

Air track with standard accessories
Air blower

Two SpeedGates incl. connection cable
Two mounting brackets for SpeedGate

Digital scales

Thread

SpeedGate

The button ***I*** selects the primary function (here: *Speed*).

The button ***II*** selects the secondary function (here: *Interval Before*).

The button ***X*** is used as on/off switch and for resetting

(Consult the SpeedGate manual for further details.)

Preparations

Setting up the air track

Place the air track on a table with one end protruding over the edge (the end without the hose).

The table should be stable and as high as possible (to allow the weight to move far).

Mount the pulley from the accessory kit in the protruding end of the track.

Connect the hose from the blower in the other end.

Equip the cart with a holder with hook in one end and a holder with a square plate in the other. For the sake of balance, always add accessories to both ends. Place a 25 mm "flag" at the top.

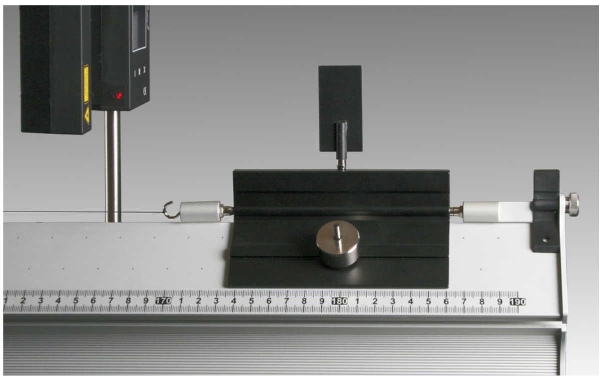
The air track must be horizontal. Turn up the air blower to allow the cart to move frictionless – not more. Adjust the track, using the three finger screws: The cart will remain motionless when the track is horizontal. Place the cart at different positions along the half of the track closest to the hose and find a suitable compromise.

The air track adjustment is *very* sensitive to any microscopic skewness – which won't affect the measurements anyway. You should concentrate on removing systematic tendencies to drift in a certain direction.

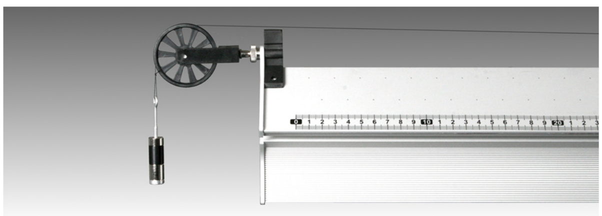
Adapting the thread

(Most easily done with the air blower turned off.)

One end of the thread is tied to the hook mounted on the cart. The other end goes over the pulley and forms a small loop for the weight.



The length of the thread is chosen to let the weight pull the cart as far as possible. This means that when the cart is flush with the end stop, the loop is just clear of the pulley.



SpeedGate setup

The two SpeedGates measures the speed of the cart while it is accelerated by the weight:

Place the first SpeedGate (A) so that the cart has moved 15 – 20 cm before the flag crosses the first light ray.

Place the second SpeedGate so that the flag has passed both light rays before the weight reaches the floor. The weight should be 1.5 – 2 cm above the floor when the flag exits the last light ray.

Insert a crossed connecting cable (included with SpeedGate) into *Chain OUT* on SpeedGate A. The other end of the cable is inserted into *Chain IN* on SpeedGate B.

Set up SpeedGate A to show *Speed* and *Previous Value*. Set up SpeedGate B to show *Speed* and *Interval Before*. In this configuration, pressing the *X* button on the first SpeedGate will reset them both.

Definitions

The speed at SpeedGate A and B: v_A and v_B

Time for passing from first to last light ray of a SpeedGate is designated similarly: t_A and t_B

Time interval between SpeedGate A and B: t_{AB}

Mass of the weight: m_W

Mass of cart, including accessories and thread: m_C

Procedure

Complete three measurement series, each with a fixed cart mass and varying pulling force.

Measurement series 1

Use the cart first *without* the cylindrical masses. (When these are added later, they are considered to be part of the cart.)

Weigh the cart including the accessories mounted and the prepared thread.

For the first measurement, use 1 small + 1 large black weight on the holder (approx. 5 g) – weigh it precisely.

Turn on the blower, pull the cart to the end stop, reset the two SpeedGates.

Release the cart without pushing. Catch it again when it has completely passed the last SpeedGate.

Read these values off the SpeedGates: Speeds at both positions and the time interval between them.

Enter the results in a table as shown below. (Use possibly a spreadsheet. The table is extended with more columns later.)

m_C =		kg	
m_W	v_A	v_B	t_{AB}
kg	m/s	m/s	s

Repeat the measurements with different masses for the pulling weight – for instance these combinations plus the holder:
small metal / small metal + all black / both metal / all.

Weigh the total mass (incl. holder) precisely every time.

Measurement series 2

Weigh two of the cylindrical weights precisely and add them to the cart. You may need to increase the air flow slightly now to keep the cart floating safely.

Repeat the measurement program from series 1. Use a new copy of the table.

Measurement series 3

Weigh and add two more cylindrical weights to the cart. Increase the air flow. Repeat the measurements again.

Theory

Measured acceleration – correction for finite time intervals

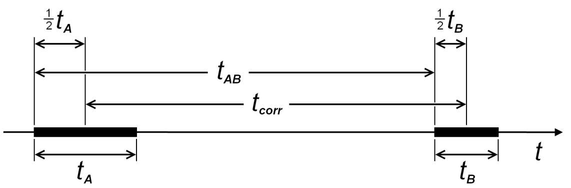
As known, acceleration is per definition

$$a = \frac{\Delta v}{\Delta t}$$

In the numerator, simply insert v_B - v_A .

One might think that t_{AB} should be substituted for Δt – but this is not quite right. The two speeds are measured in time intervals t_A and t_B which doesn't lie symmetrically around the start and finish times for t_{AB}. This leads to a systematic error. Instead, Δt must be found as the difference between the *centres* of the two time spans t_A and t_B – see figure.

This way we are led to introduce the corrected time interval t_{corr} .



As can be seen from the figure, this can be expressed

$$t_{\text{corr}} = t_{\text{AB}} + \tfrac{1}{2}t_{\text{B}} - \tfrac{1}{2}t_{\text{A}}$$

– or by measured entities alone:

$$t_{\text{corr}} = t_{\text{AB}} + \frac{\Delta s}{2 \cdot v_{\text{B}}} - \frac{\Delta s}{2 \cdot v_{\text{A}}}$$

Δs = 2.00 cm is the distance between the light rays.

Finally we can calculate the acceleration:

$$a_{\text{meas}} = \frac{v_{\text{B}} - v_{\text{A}}}{t_{\text{corr}}}$$

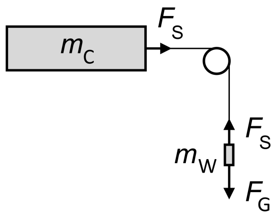
Forces and masses – Newton's second law

The cart and the weight accelerate equally fast as the length of the thread is constant.

The net force on the cart is the string force. The net force on the weight is the force of gravity minus the string force – see figure.

Assuming Newton's second law to be valid, we get these relations:

$$F_{\text{S}} = m_{\text{C}} \cdot a$$
$$F_{\text{G}} - F_{\text{S}} = m_{\text{W}} \cdot a$$



Inserting the first equation into the second we get

$$F_{\text{G}} = (m_{\text{C}} + m_{\text{W}}) \cdot a \qquad (1)$$

Which can be interpret as Newton's second law for a system with total mass m_C + m_W , accelerated by a net force F_G , the force of gravity on the weight.

Looking closer at the system, you discover one more moving part: the *pulley*. The mass of the pulley is also accelerated – but not completely like the cart and the weight: The rim moves with the same speed as the cart and the weight but the rest of the pulley moves slower the closer you get to its centre.

It is possible to reduce the mass distribution of the pulley as if it was placed only along the rim. (The details fall outside the scope of this manual.) We will call the result the *equivalent mass* of the pulley and designate it m_P*.

The pulley from the accessory box has m_P* = 2,43 g.

Equation (1) must therefore be extended like this:

$$F_{\text{G}} = (m_{\text{C}} + m_{\text{W}} + m_{\text{P}}^{*}) \cdot a \qquad (2)$$

The force of gravity on the weight can also be found from its mass and the acceleration due to gravity:

$$F_{\text{G}} = m_{\text{W}} \cdot g \qquad (3)$$

Equating the two expressions for F_G (2) and (3), we get the theoretical value of the acceleration as predicted by Newton's second law:

$$a_{\text{theo}} = \frac{m_{\text{C}}}{m_{\text{C}} + m_{\text{W}} + m_{\text{P}}^{*}} \cdot g$$

Calculations

Extend the table with these columns:

	t_{corr}	a_{meas}	a_{theo}	dev.
	s	m/s²	m/s²	%

The last column is the relative deviation between measured and theoretical acceleration.

Discussion and evaluation

To demonstrate Newton's second law, its predictions must correspond to what we measure – within the limits of measurement uncertainties.

The uncertainty of one speed measurement is 1 % – the uncertainty on a_{meas} is somewhat larger.

The relative uncertainty on a_{theo} is approx. the same as that on m_L .

Do your measurements substantiate Newton's second law – or are there deviations that cannot be explained?

Consider possible sources of error.

Teacher's notes

Concepts used

Momentary speed
Average speed
Acceleration
Net force
Gravity

Mathematical skills

Percent
Evaluation of expressions
(Spreadsheet)

About the equipment

The cart weighs a little more than 200 g including accessories, but without the cylindrical weights. By weighing the cylindrical weights separately and adding their masses to the mass of the cart you can still use inexpensive but quite accurate scales like the 102900 mentioned in the equipment list.

SpeedGates measure the passage time based on the front of the object crossing the light rays. The width of the flag on the cart is therefore unimportant.

The air track must be adjusted horizontally before the start of the measurements. The students can do this themselves – or it can be completed before the lab session.

The air blower must be turned up until the cart is sure to float freely. On the other hand, the air pressure should not be unnecessarily high as the individual air jets may tilt the cart slightly which will result in a horizontal force component.

The theory section

In the section *Theory* only hand-waving arguments are offered when introducing t_{corr}. For motion with constant acceleration, the derivation can be made exact. This would require a bit more columnage.

The introduction of "equivalent mass" of the pulley is based on a calculation of the moment of inertia of the pulley. From this, it is easy to calculate the mass that results in the same moment of inertia if distributed along the rim. The factors of r that are included in the moment of inertia, the torque and the angular acceleration can then be eliminated in the equation of motion for the pulley. Hence the equivalent mass can be considered to undergo translational acceleration.

Detailed equipment list

Specifically for the experiment

195050	Air track (incl. accessories)
197070	Air blower
197570	SpeedGate (Qty. 2)
195055	Mounting bracket for 197570 (Qty. 2)

Standard lab equipment

102900	Digital scales, 300 g / 0,01 g – or similar

If an older air track is used (e.g. 195000) – replace the two 195055 by this stand material:

000100	Retort stand base, 2,0 kg (Qty. 2)
000830	Retort stand rod 50 cm (Qty. 2)
002310	Square bosshead (Qty. 2)

Consumables and spare parts

116500	Extra strong thread
197571	Cable modular crossed 2m (This cable is included with a SpeedGate)