

Experiment number	135110-EN	Topic	Mechanics		
Version	2016-06-14 / HS	Type	Student exercise	Suggested for grade 10-12	p. 1/4

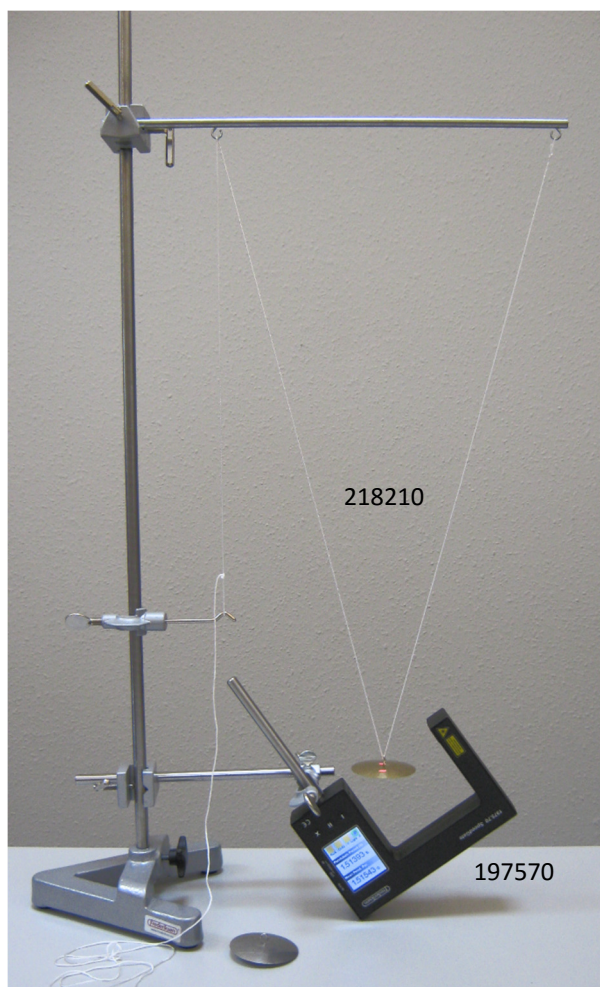
## Objective

Establishing the formula for the oscillation period of a mathematical pendulum.

## Principle

A mass of small extent, which oscillates in a light thread is a good approximation of a so-called *mathematical pendulum*. (Ideally, it is a point mass in a rigid, massless string with frictionless suspension - in vacuum.)

In order to check the formula for the oscillation period, the different parameters are varied – both some that are part of the formula, and others that should not have any influence. The two weights included have the same shape but different mass. The pendulum length and the amplitude of the oscillation can also be varied.



Stand material is listed on page 4

## Equipment

(Detailed equipment list on the last page.)

218210 Mathematical pendulum w. suspension

197570 SpeedGate

Stand material

### Mathematical pendulum 218210

The pendulum bob hangs in a double thread that ensures that the direction of the swing is fixed.

The shape of the weights concentrates the mass at a certain distance from the axis of oscillation. In addition the air drag is minimal.

### SpeedGate 197570

Photogate with a built-in display. Can among other things measure period.

With a pendulum, you typically use the *Pendulum Period* mode which drops every other interception of the light ray.

**Procedure**

The setup is shown on page 1 (details to the right).

Make a fixed loop at one end of the thread.

The loop is hooked on the outermost hook at the suspension rod. From there, the thread runs down to the weight, up to the other hook and is fixed at a stand clamp with hook.

SpeedGate is turned on and reset by the *X* button. Mode of operation is selected by buttons *I* and *II*.

Select *Pendulum Period* and *Mean Pendulum Period*. Measuring the period goes like this:

1. Start the oscillation
2. Press *Reset (X)*
3. Read the mean period *T* after a sufficient number of oscillations – e.g. five

The weight is so slim that it causes problems if it was to block a horizontal light ray. This is solved by tilting the SpeedGate – then the edge of the weight cuts the light ray reliably.

Adjust the mutual positions of the suspension and the SpeedGate in order to let light ray “X” hit the centre of the weight when it hangs at rest.

You will among other things measure *the length of the pendulum*. This is **not** just the length of the thread. We consider a line through the two points of suspension to be an axis of rotation and define the length *L* of the pendulum as the vertical distance between this axis and the centre of the weight.

To ensure a fixed amplitude, you can place an extra retort stand to mark the position from where the pendulum bob is started.

**1 – Fixed mass and amplitude; varying length**

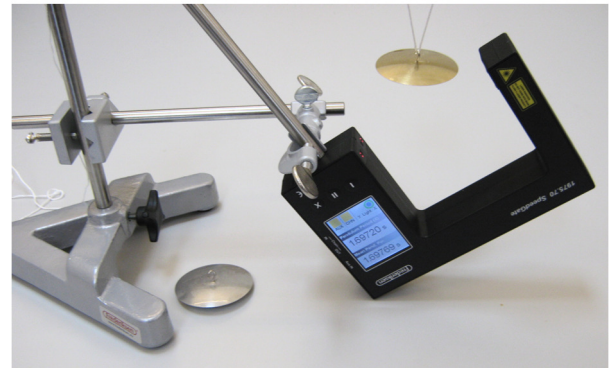
In this part we only use the brass weight. Measure the period for at least 5 different lengths between approx. 30 cm and 1 m.

Each time, *L* is measured precisely – preferably with a precision of 1 mm. Write down the length in metres.

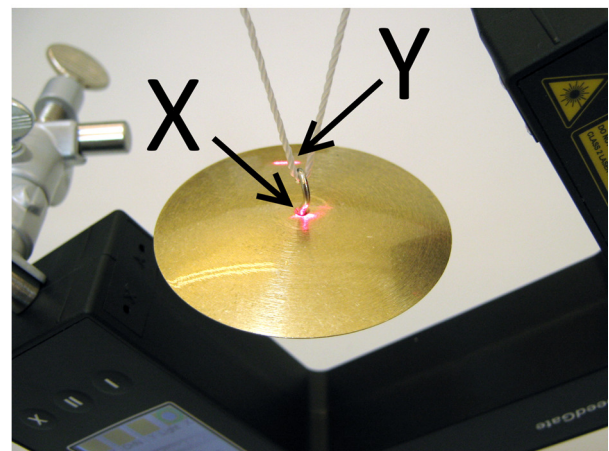
Use a moderate amplitude; the bob should only swing a few centimetres away from light ray “X”.

Record the results in a table like this:

Mass of the bob / g	
Amplitude / cm	
<i>L</i> / m	<i>T</i> / s

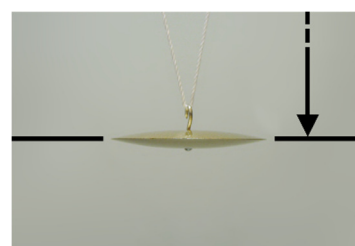
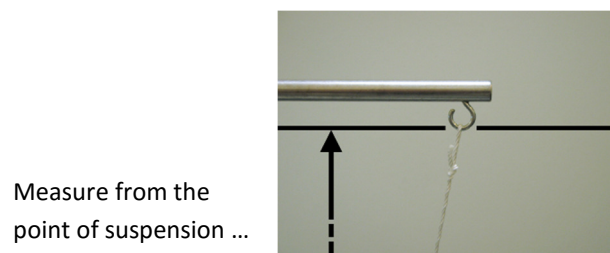


SpeedGate is tilted to ensure that the pendulum bob with certainty blocks the light ray.



When the pendulum hangs vertical, the X light ray should hit the bob at the centre. (The Y ray hits close to the edge of the weight).

The length of the pendulum *L* (Measured **vertically**) :



... to the centre of the weight.

## 2 – Fixed length and amplitude; varying mass

This part of the experiment must – like the previous – be performed with a rather small amplitude.

The length can e.g. be chosen as approx. 0.75 m.

The masses of the weights need only be determined with a precision of 0.1 to 1 g.

Write down the results in a table like this:

Length $L$ / m	
Amplitude / cm	
$m$ / g	$T$ / s

## 3 – Fixed mass and length; varying amplitude

Use the brass weight in this part. The length of the pendulum can be unchanged from the previous part.

As described earlier, an extra retort stand can be used as a start position for the weight. The distance from this stand to the edge of the weight (hanging at rest) is a measure for the amplitude.

(Often amplitude is instead defined as the maximum *angle* from the vertical. This is more difficult – and the difference has no impact on the outcome of this experiment.)

Vary the amplitude from a few centimetres until the thread is almost horizontal. 5 to 10 measurements is fine.

Keep an eye on the direction of the swing: The bob must not hit the SpeedGate! Start eventually behind the SpeedGate.

Results are entered in a table like this:

Mass of the bob / g	
Length $L$ / m	
Amplitude $A$ / cm	$T$ / s

## Theory

The period of oscillation of a mathematical pendulum swinging with a small amplitude is

$$T = 2 \cdot \pi \cdot \sqrt{\frac{L}{g}}$$

where  $g$  is the acceleration due to gravity (table value).

We notice that out of the parameters we are varying only the pendulum length is part of the formula.

The relation may be rewritten like this:

$$T^2 = \frac{4 \cdot \pi^2}{g} \cdot L$$

This means that if  $T^2$  is plotted as a function of  $L$ , the result will be a straight line through (0,0) with a slope given by  $\frac{4 \cdot \pi^2}{g}$ .

## Calculations etc.

### 1 – Varying length

For each measurement, calculate  $T^2$  and plot  $T^2$  as a function of  $L$ .

Draw the best straight line through the data points and (0,0). Find the slope of the line.

### 2 – Varying mass

Calculate as a percentage the increase in mass when going from the aluminium weight to the brass one.

Calculate as a percentage the increase in the period, going from the aluminium weight to the brass one.

### 3 – Varying amplitude

Plot  $T$  in a coordinate system as a function of  $A$ .

## Discussion and evaluation

Describe the consistency between theory and measurements for the first part.

How does the results of part 2 agree with the theory that claims the period to be independent of the mass? Compare with how constant the other parameters can be kept. Does the thread give? Are the hooks on the weights of exactly the same shape?

Evaluate the prediction of the theory, that the period does *not* depend on the amplitude if only this is kept “small”.

## Teacher's notes

### Concepts used

Period of oscillation

### Mathematical skills

Plotting graphs

Slope of a straight line

Percent

### About the equipment

SpeedGate measures time with millisecond resolution. You cannot expect the remaining experimental circumstances to be so well defined that all decimals of the period measurements are meaningful.

The high resolution of the SpeedGate makes it easy to see that the period changes for each swing when the amplitude is large. This is due to friction in the suspension and air drag. You may want to reduce the number of periods used in averaging in part 3.

### Didactic considerations

In this lab manual, students are encouraged to assess the agreement between theory and measurements in several ways – but never by numerical comparison, data point by data point. Should you prefer this approach, just rephrase the last section of the manual.

## Detailed list of equipment

### Specifically for the experiment

218210 Mathematical pendulum w. suspension  
197570 SpeedGate

### Standard lab equipment

000100 Retort stand base, A-shaped, 2.0 kg  
000800 Retort stand rod, 150 cm  
000850 Retort stand rod, 25 cm  
002310 Square boss head (2 are used)  
002320 Boss head, swivel  
002700 Stand clamp with hook  
140510 Ruler, wood 100 cm  
102961 Digital scale 200 g/0,1 g – or similar