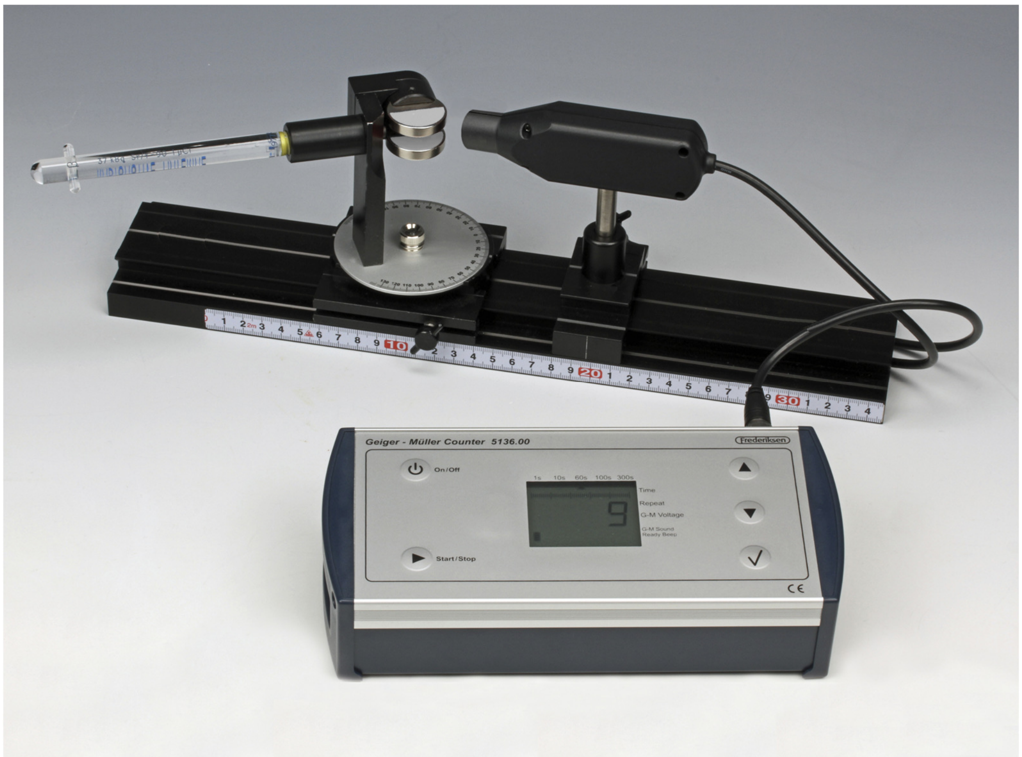
The beta spectrum (advanced version)

Number	138550-EN	*Topic*	Radioactivity, motion of charged particles				
Version	2017-09-14 / HS	*Type*	Student exercise	*Suggested for*	grade 12+	*p.*	1/4



Objective
To investigate the energy distribution of beta radiation. The maximum energy of the radiation is found. The experimental and the theoretical spectrum are plotted and compared.

Principle
The radiation is collimated by a plastic aperture. After that, it passes an area with a strong magnetic field from a pair of permanent magnets. In the magnetic field the trajectory of the beta particles is circular with a radius that depends on the velocity of the particles.

The deflection angle is read on the apparatus and is converted into kinetic energy with the help of a graph.

Equipment
(Detailed equipment list on p. 4.)

Deflection of beta particles
Experiment bench or rail
Beta source (Risø) *

Geiger-Müller tube
Geiger counter
(Alternative means of counting may be employed)

Bar magnet with known polarity

If possible a teslameter

* Other types of source can be used. (See p. 4.)

Work carefully
Follow your teacher's instructions for working with radioactive sources.

Keep a suitable distance to the sources
Limit the time you need to handle or stay close to the sources
Consumption of food or beverages is not allowed in the room while the sources are used

Sources with a handle should only be manipulated using the end that is furthest away from the source.

Frederiksen Scientific A/S
Viaduktvej 35 · DK-6870 Ølgod

Tel. +45 7524 4966
Fax +45 7524 6282

info@frederiksen.eu
www.frederiksen.eu

Theory

The appearance of the beta spectrum

In 1933, Enrico Fermi developed the theory of beta decay, and was able to account for the shape of the spectrum. The frequency of beta particles emitted with kinetic energy in the range dE around the value E is given by

$$N_{\text{theo}}(E)\cdot \mathrm{d}E = C\cdot\sqrt{E^2+2E\cdot m_0c^2}\cdot(E_{\max}-E)^2\cdot(E+m_0c^2)\cdot F(Z',E)\cdot\mathrm{d}E$$

where $F(Z',E)$ is the *fermi function,* describing the influence of the electrostatic attraction of the beta particles towards the nucleus. Z' is the atomic number of the daughter nucleus, m_0 is the rest mass of the electron, c is the speed of light in vacuum. (C is just a constant.)

For Z less than about 50, $F(Z,E)$ can be approximated by the expression

$$F(Z,E)=\frac{2\cdot\pi\cdot\nu}{1-\mathrm{e}^{-2\cdot\pi\cdot\nu}}\left(\alpha^2\cdot Z^2\cdot\omega^2+\tfrac{1}{4}(\omega^2-1)\right)^S$$

where $\alpha = 1/137$ is called the fine structure constant and the other entities are:

$$\nu=\alpha\cdot Z\cdot\frac{E+m_0c^2}{\sqrt{E^2+2\cdot E\cdot m_0c^2}}\quad,\qquad \omega=\frac{E}{m_0c^2}+1\quad,\qquad S=\sqrt{1-\alpha^2Z^2}-1$$

Kurie plot

At higher energies, the fermi function is by and large constant and the expression for $N(E)$ can be linearized: If you plot the value

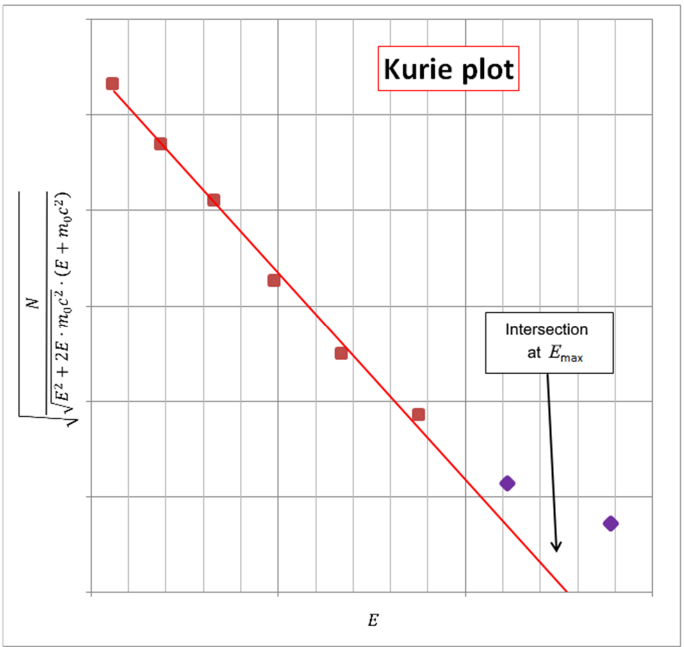
$$K(E)=\sqrt{\frac{N(E)}{\sqrt{E^2+2E\cdot m_0c^2}\cdot(E+m_0c^2)}}$$

as a function of E, the result will be a straight line intersecting the x axis at $E_{\max}$. This is called a Kurie plot.

When using this equipment (514105), uncertainties are largest for high energies

As can be seen on the graph you may have to reject a few of the data points around $E_{\max}$ which clearly deviates from a straight line (the two purple points).

After that you should be able to reach a good agreement with the expected value..



The energy of the beta particles

To establish the relation between the deflection angle θ and the kinetic energy E, we will assume that the magnetic field is homogeneous with flux density B between the magnets – and zero outside. The radius of the magnets is called R. m_0, e and c designate the rest mass of the electron, the elementary charge resp. the speed of light. The relation can be shown to be

$$E=m_0c^2\cdot\left(\sqrt{\left(\frac{e\cdot B\cdot R}{m_0\cdot c\cdot\tan\left(\frac{\theta}{2}\right)}\right)^2+1}-1\right)$$

The assumptions about the geometry of the magnetic field are not quite fulfilled but if you allow yourself to adjust B or R slightly, acceptable agreement with the theoretical spectrum can be achieved.

Correcting for measurement interval distribution

If measured in equally large energy intervals, you will expect proportionality between measured and theoretical spectra. In reality, count numbers with this equipment aren't measured in fixed **energy** intervals but instead in fixed **angle** intervals (primarily given by the geometry of the collimator, the GM tube size and the distance to the magnet centre).

As $E(\theta)$ isn't a linear function, the counts must be divided by $|\mathrm{d}E/\mathrm{d}\theta|$, given by

$$\left|\frac{\mathrm{d}E}{\mathrm{d}\theta}\right|=\frac{1}{2}\cdot m_0c^2\cdot\frac{b^2}{\sin^2\left(\frac{\theta}{2}\right)\cdot\sqrt{b^2+\tan^2\left(\frac{\theta}{2}\right)}}$$

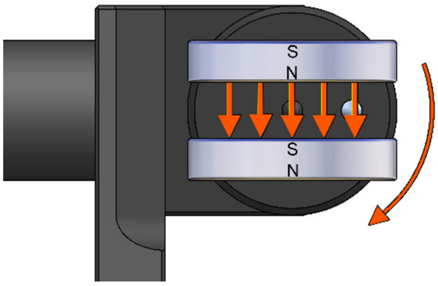
where

$$b=\frac{e\cdot B\cdot R}{m_0\cdot c}$$

Procedure

Setup: See p. 1. Distance between magnets and GM tube should be approx. 10 mm.

Turn the magnets completely **clockwise** to make the field point straight down (see figure).



North – red marking – down. The polarity of the magnet can be checked with a bar magnet.

The magnet assembly should be mounted during the whole experiment.

If a teslameter is available: Measure the B field at the centre. If not: assume the value 310 mT.

Measure the diameter of the magnets.

Vary the angle θ between approx. 45° and 140° in 5° steps and record the count N for each angle, using for instance 100 second periods.

Measure the background radiation N_0 for the same period with the source removed far from the GM tube.

Calculations etc.

Kurie plot – determining E_{max}

Make a table as shown. Use a spreadsheet program.

Insert all measured data. For the Kurie plot we only use data for the smallest angles (i.e. the highest energies) up to about the maximum value of N.

Measured		Calculated			
θ	N	E	$\lvert dE/d\theta \rvert$	N'	$K(E)$
°		keV	-		-

The kinetic energy of the beta particles is found from the deflection angel as shown in the theory paragraph.

Calculate the factor $\lvert dE/d\theta \rvert$ for each angle.

First correct the counts for background radiation, next correct the result to "per energy" for each angle by dividing by $\lvert dE/d\theta \rvert$. Call the result N'.

For the Kurie plot, calculate $K(E)$. The count to use in the formula are N' (not N).

(As long as you use *consistent* units of measure all over, we will not require explicit units for the last three columns.)

Finding E_{max}:
Make a Kurie plot based on the data points for high energies. Drop the *very* highest which has large uncertainties.
From the Kurie plot the maximum beta energy for this type of source can be found.

The spectrum

Add columns to the previous table. In the upper line, C denotes the constant of proportionality for scaling the theoretical spectrum $N_{teo}(E)$. For calculating N_{teo} you may use the table value for E_{max}.

Uncertainties: $\Delta E = \lvert dE/d\theta \rvert \cdot \Delta\theta$ (assume $\Delta\theta$ = 5°).

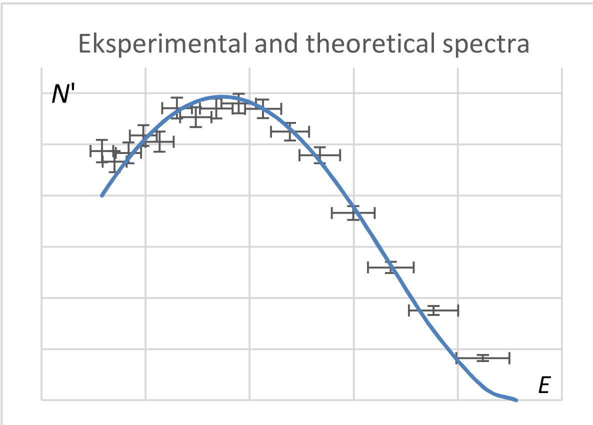
$$\Delta N' = \frac{\sqrt{N}}{\lvert dE/d\theta \rvert}$$

All other terms are from the theory paragraph. It is acceptable to drop the units of ω, ν and F.

$S(Z)$=		E_{max}=		C =	7,6E-12
ΔE	$\Delta N'$	ω	ν	$F(Z',E)$	$N_{theo}(N)$
keV		-	-	-	

Draw the spectrum:
Plot the corrected counts as a function of the kinetic energy. Remember units along the axes. Draw uncertainties ("error bars") for both coordinates.

Add the theoretical spectrum to the graph. Adjust C manually until best agreement is achieved. It may be necessary to drop data point at the lowest energies.



Example of spectra (units removed)

Discussion and evaluation

Compare the E_{max} found with the table value.

Discuss the agreement between the experimental and the theoretical spectrum.

Try to relate any deviations to the equipment and indicate possible improvements. Assume wide (but not astronomical) economical limits.

Teacher's notes

Concepts used

Decay
Registration of ionising radiation
Background radiation
Atomic number
Energy
Charged particles in a magnetic field

Mathematical skills

Calculus
Evaluation of complicated expressions
Advanced graph use in a spreadsheet

About the equipment

The beta source is a Sr/Y-90 source. The radiation from the Sr-decay has a rather low maximum energy and cannot be examined with this equipment. It is the decay from Y-90 that is investigated.

Didactic considerations

(Part of) the theory can be stated as given facts – but some of the formulae could be deduced by the students as a part of the report.

Types and availability of sources

Frederiksen Scientific cannot provide sources unless we receive documentation that the customer and the end user are entitled to handling and using such sources.

Frederiksen Scientific only provides sources of the "Risø" type – seen on the photo on p. 1 – but we make equipment that is compatible with two other widely used types:

Disc-shaped (Ø 25 mm) sources
Cylindrical (Ø 12 mm) sources

The nuclide used is detailed in the "About the equipment" section.

It must be noted that the Sr/Y-90 source must be specifically constructed for beta emission.

Detailed equipment list

Specifically for Risø sources

510020	Beta source (Included with 510000 Risø sources, complete set)
514105	Deflection of beta particles (Risø source)

Specifically for disc sources

	Beta source (disc) as described above
514125	Deflection of beta particles (disc source)

Specifically for cylinder sources

	Beta source (cylindrical) as described above
514135	Deflection of beta particles (cyl. source)

Independent on source design

514102	Rail for experiment bench, 40 cm (Included with the 514100 Exp. Bench)
294610	Saddle with Ø10mm hole (Included with the Experiment Bench)
330850	Bar magnets, pair
513610	Geiger counter (or similar)
512515	Geiger-Müller tube with BNC-plug
406050	Teslameter (Optional – omitted if optimum precision is not needed)

Alternative

Although our complete experiment bench (including absorber plates and a saddle) is not needed in this experiment, it constitutes a versatile base for several experiments with radioactivity. You can substitute the two items 514102 and 294610 with

For Risø sources:	514100
For disc sources:	514120
For cylindrical sources:	514110